Stepwise Synthesis of Partial Specifications preserving Strong (Ω_1,Ω_2)-Equivalence(Concurrency Theory and Applications '96)

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Stepwise Synthesis of Partial Specifications preserving Strong (Ω_1, Ω_2)-Equivalence

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abstract : In this paper we present a *partial* equality called *strong* (Ω_1, Ω_2)-*equivalence* between two partial specifications SP_1 and SP_2, denoted by $SP_1 \ {}_{\Omega_1}\!\sim_{\Omega_2} SP_2$, based on an extended bisimulation relation. The parameters Ω_1 and Ω_2 are the sets of available actions in SP_1 and SP_2, respectively. Furthermore we present a synthesis method of the two partial specifications SP_1 and SP_2 into a specification SP_{12} such that $SP_{12} \ {}_{\Omega_1\cup\Omega_2}\!\sim_{\Omega_1} SP_1$ and $SP_{12} \ {}_{\Omega_1\cup\Omega_2}\!\sim_{\Omega_2} SP_2$. SP_{12} is called the *principal strong* (Ω_1, Ω_2)-*synthetic specification* of SP_1 and SP_2, and it is denoted by $SP_{12} \doteq (SP_1 \ {}_{\Omega_1}\!\lceil\!\rceil_{\Omega_2} SP_2)$. For example, this synthesis method is used for stepwise refinement of partial specifications preserving strong (Ω_1, Ω_2)-equivalence.

1 Introduction

A specification of a large system is generally too complex for users (and also designers) to understand, and users are often interested in *some* actions. Therefore it is useful for users to extract partial specifications about their interesting actions from the complete specification by hiding uninteresting actions. On the other hand, it is also needed to stepwise synthesize partial specifications required by many designers.

In this paper, we propose a partial analysis method in models based on labeled transition systems (LTS). A set of interesting actions is called a *filter* ranged over by Ω. An action observed through the filter is changed into an internal action τ, if the action is not included in the filter.

We explain the partial analysis by using Figure 1. The transition graph SP shows the specification of a system SYS. Each node represents a state and each labeled arrow represents a transition by the label which corresponds to an action. In this paper we use the sequential process expressions of a fundamental process algebra CCS[1] for describing specifications as follows:

$$SP \stackrel{\text{def}}{=} a.(b.c.SP + b.\tau.SP) + a.(b.\tau.SP + \tau.c.SP)$$

where the period '.' is a sequential combinator and '+' is a choice combinator. $\stackrel{\text{def}}{=}$ is used for defining the left side *specification constant* as the right side specification, thus it is a recursive definition.

SP_1 is a partial specification of SYS observed through the filter $\{a, b\}$. In this case the two states (3) and (4) in SP are not distinct in SP_1, because c is changed into τ through $\{a, b\}$. Although it is expected that two sequential transitions by τ from (2) to (0) via (5) in SP_1 is reduced to one transition by τ, the reduction is not considered in this paper. The reduction of internal actions based on weak equivalence of CCS is the next work, and we first clarify

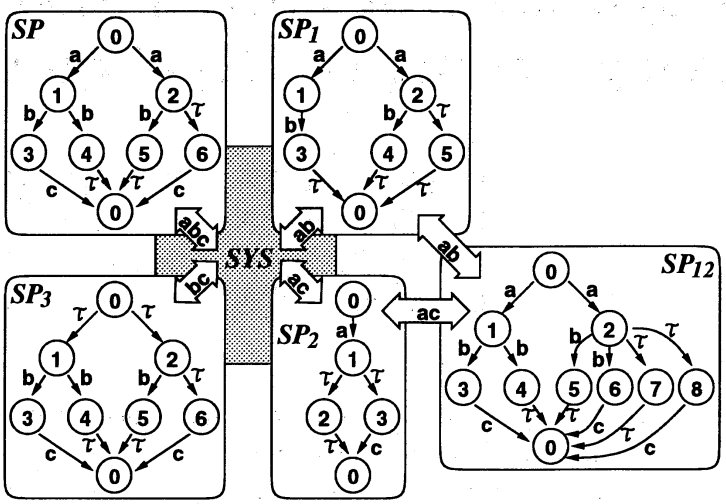

Figure 1: Partial specifications (SP_1, SP_2, SP_3) and a synthetic specification SP_{12}

the effect of the filter based on strong equivalence. SP_2 and SP_3 are partial specifications observed through $\{a, c\}$ and $\{b, c\}$, respectively.

We give a *partial* equality called *strong* (Ω_1, Ω_2)*-equivalence* denoted by ${}_{\Omega_1}\sim_{\Omega_2}$, for relating such partial specifications, where Ω_1 and Ω_2 are filters used for getting the left side partial specification and the right side one, respectively. For example, SP_1 and SP_2 are strongly $(\{a, b\}, \{a, c\})$-equivalent and it is denoted by

$$SP_1 \ {}_{\{a,b\}}\sim_{\{a,c\}} \ SP_2.$$

Braces and commas of filters are often omitted such that $\{a, b\}$ is written as ab.

Next, we discuss how to reconstruct the complete specification by synthesizing partial specifications. For the example of Figure 1, it is expected to produce a synthetic specification SP_{12} such that

$$SP_{12} \ {}_{abc}\sim_{ab} \ SP_1 \quad \text{and} \quad SP_{12} \ {}_{abc}\sim_{ac} \ SP_2. \qquad (*_1)$$

In this case, it is a problem that there are many synthetic specifications satisfying $(*_1)$ and they are always not strongly (abc, abc)-equivalent. For example, the following specifications satisfy $(*_1)$, but $SP_{12} \ {}_{abc}\not\sim_{abc} SP'_{12}$.

$$SP_{12} \stackrel{\text{def}}{=} a.(b.c.SP_{12} + b.\tau.SP_{12}) + a.(b.\tau.SP_{12} + \tau.c.SP_{12} + b.c.SP_{12} + \tau.\tau.SP_{12})$$
$$SP'_{12} \stackrel{\text{def}}{=} a.(b.c.SP'_{12} + b.\tau.SP'_{12}) + a.(b.c.SP'_{12} + \tau.\tau.SP'_{12})$$

It is also important to notice that $SP_{12} \ {}_{abc}\not\sim_{bc} \ SP_3$ and $SP'_{12} \ {}_{abc}\not\sim_{bc} \ SP_3$. This inequality complicates the next synthesis of SP_{12} and SP_3, thus stepwise synthesis of specifications is difficult.

In order to overcome this problem, we attempt to describe a *comprehensive* specification which includes all the synthetic specifications satisfying $(*_1)$. Thus a synthetic specification satisfying $(*_1)$ can be selected form the comprehensive specification. For describing the comprehensive specification, we present a *multi-labeled transition system* abbreviated to a *multi-LTS*. A transition of the multi-LTS has the following form

$$P \xrightarrow{\langle \alpha_1, \alpha_2, \cdots, \alpha_n \rangle} \langle P_1, P_2, \cdots, P_n \rangle$$

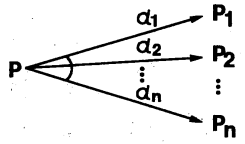

Figure 2: A transition in the multi-labeled transition system (multi-LTS)

and its transition graph is written as shown in Figure 2. It intuitively means that P can perform an action α_1 then behaves like P_1, P can perform an action α_2 then behaves like P_2, $\cdots$, *and/or* P can perform an action α_n then behaves like P_n. The '*and/or*' is important. For example, the following specification AB

$$AB \xrightarrow{\langle a,b\rangle} \langle \mathbf{0}, \mathbf{0}\rangle$$

can perform a then stops and/or can perform b then stops.

We present a specification language $SPEC^{\vee}$ based on the multi-LTS. $SPEC^{\vee}$ consists of the sequential combinators of CCS and a new combinator $\vee$ called an *Andor combinator*. Intuitively $P \vee Q$ behaves P or Q or $P + Q$. We often say that $P \vee Q$ *includes* P, Q, and $P + Q$. For example, the above AB can be described as follows:

$$AB \stackrel{\text{def}}{=} (a.\mathbf{0}) \vee (b.\mathbf{0})$$

The important difference between $\vee$ and $+$ is explained as follows:

- $(a.\mathbf{0}) \vee (b.\mathbf{0})$ can perform a and/or b.
- $(a.\mathbf{0}) + (b.\mathbf{0})$ can perform a and b.

If a specification contains no Andor combinators, then it is called a *ground specification*. Thus ground specification can be described in CCS. For example, $(a.\mathbf{0}) + (b.\mathbf{0})$ is a ground specification. A specification of a practical system is always a ground specification. Specifications containing Andor combinators are used for describing medium specifications during design process.

Strong (Ω_1, Ω_2)-equivalence previously introduced for ground specifications is extended to $SPEC^{\vee}$. Intuitively, two specifications P and Q are strongly (Ω_1, Ω_2)-equivalent, denoted by $P \ {}_{\Omega_1}\!\sim_{\Omega_2} Q$, if $P_0 \ {}_{\Omega_1}\!\sim_{\Omega_2} Q_0$ for *some* ground specifications P_0 and Q_0 included in P and Q, respectively. For example,

$$(a.\mathbf{0}) \vee (b.\mathbf{0}) \ {}_{abc}\!\sim_{abc} (a.\mathbf{0}) \vee (c.\mathbf{0}),$$

because both of them include a ground specification $(a.\mathbf{0})$.

Another equality called *strong* (Ω_1, Ω_2)*-full-equivalence* is also given. Intuitively, two specifications P and Q are strongly (Ω_1, Ω_2)-full-equivalent, denoted by $P \ {}_{\Omega_1}\!\simeq_{\Omega_2} Q$, if $P_0 \ {}_{\Omega_1}\!\sim_{\Omega_2} Q_0$ for *any* ground specifications P_0 and Q_0 included in P and Q, respectively. For example,

$$(a.\mathbf{0}) \vee (b.\mathbf{0}) \ {}_{abc}\!\simeq_{abc} (b.\mathbf{0}) \vee (a.\mathbf{0}),$$
$$(a.\mathbf{0}) \vee (b.\mathbf{0}) \ {}_{abc}\!\not\simeq_{abc} (a.\mathbf{0}) \vee (c.\mathbf{0}),$$
$$(a.\mathbf{0}) \vee (b.\mathbf{0}) \ {}_{abc}\!\not\simeq_{abc} ((a.\mathbf{0}) \vee (b.\mathbf{0})) + (a.\mathbf{0}).$$

The reason of the last inequality is that $(a.\mathbf{0}) \vee (b.\mathbf{0})$ includes a ground specification $(b.\mathbf{0})$ but $((a.\mathbf{0}) \vee (b.\mathbf{0})) + (a.\mathbf{0})$ does not include $(b.\mathbf{0})$.

Then we present a relation called the *principal strong* (Ω_1, Ω_2)*-synthetic specification* P of two specifications Q_1 and Q_2, denoted by $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\!\rceil_{\Omega_2} Q_2)$. For the example of Figure 1,

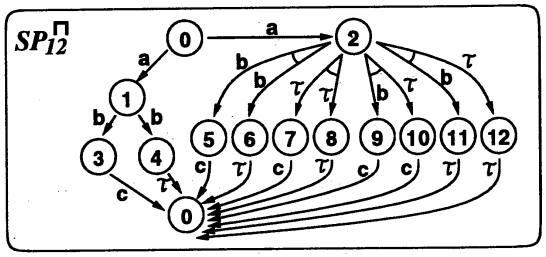

Figure 3: The transition graph of $SP_{12}^{\sqcap}$

the following specification $SP_{12}^{\sqcap}$ is the principal strong (ab, ac)-synthetic specification of SP_1 and SP_2,

$$\begin{aligned} SP_{12}^{\sqcap} \stackrel{\text{def}}{=} & a.(b.c.SP_{12} + b.\tau.SP_{12}) \\ & + a.((b.c.SP_{12} \vee b.\tau.SP_{12}) + (\tau.c.SP_{12} \vee \tau.\tau.SP_{12}) \\ & + (b.c.SP_{12} \vee \tau.c.SP_{12}) + (b.\tau.SP_{12} \vee \tau.\tau.SP_{12})), \end{aligned}$$

thus $SP_{12}^{\sqcap} \doteq (SP_1 \,{}_{ab}\lceil\rceil_{ac}\, SP_2)$. Figure 3 shows the transition graph of $SP_{12}^{\sqcap}$. In Section 4 it is shown how to check whether a specification is the principal strong (Ω_1, Ω_2)-synthetic specification of two specifications, and how to produce the principal strong (Ω_1, Ω_2)-synthetic specification from two specifications. The principal strong (Ω_1, Ω_2)-synthetic specification is *uniquely* decided.

The main property of the principal strong (Ω_1, Ω_2)-synthetic specification is shown in Proposition 4.1. For the example of Figure 1, the following relation holds by Proposition 4.1. For any ground specification SP_0,

$$SP_0 \,{}_{abc}\!\sim_{abc} SP_{12}^{\sqcap} \quad \textbf{iff} \quad SP_0 \,{}_{abc}\!\sim_{ab} SP_1 \quad \text{and} \quad SP_0 \,{}_{abc}\!\sim_{ac} SP_2.$$

Thus, all the ground specifications included in $SP_{12}^{\sqcap}$ satisfy both SP_1 and SP_2, and they are all the ground specifications satisfying both SP_1 and SP_2. Furthermore SP_3 can be synthesized to $SP_{12}^{\sqcap}$ by $SP_{123}^{\sqcap} \doteq (SP_{12}^{\sqcap} \,{}_{abc}\lceil\rceil_{bc}\, SP_3)$. Then the following relation holds by Theorem 4.4, because $SP_{12}^{\sqcap} \,{}_{abc}\!\sim_{bc} SP_3$. For any ground specification SP_0,

$$SP_0 \,{}_{abc}\!\sim_{abc} SP_{123}^{\sqcap} \textbf{ iff } SP_0 \,{}_{abc}\!\sim_{ab} SP_1,\ SP_0 \,{}_{abc}\!\sim_{ac} SP_2, \text{ and } SP_0 \,{}_{abc}\!\sim_{bc} SP_3.$$

Thus stepwise refinement of specifications is possible.

The outline of this paper is as follows: In Section 2, we define the syntax and the semantics of $SPEC^{\vee}$. In Section 3, strong (Ω_1, Ω_2)-equivalence is defined. In Section 4, principal strong (Ω_1, Ω_2)-synthetic specifications are defined. In Section 5, we discuss partial analysis and synthesis methods already proposed. In Section 6, we conclude this paper.

2 Definition of specifications

We use expressions like CCS for describing behavior of specifications. They are sequential processes of CCS with Andor combinators introduced in Section 1. We call the specification language $SPEC^{\vee}$. In this section, we define the syntax and the semantics of $SPEC^{\vee}$.

We assume that an infinite set of *names* $\mathcal{N}$ ranged over by $a, b, \cdots$, is given. The set of actions *Act* ranged over by $\alpha, \beta, \cdots$, is defined as $Act = \mathcal{N} \cup \{\tau\}$, where τ is a special action called an *internal action* not included in $\mathcal{N}$. We also assume that a set of *specification constants* (also called *Constants*) $\mathcal{K}$ ranged over by $A, B, \cdots$, is given. Then, the syntax of $SPEC^{\vee}$ is defined.

Definition 2.1 *The set of specifications $\mathcal{P}$ is the smallest set including the following expressions:*

$$\begin{array}{rcl} A & : & \textit{Constant } (A \in \mathcal{K}) \\ \mathbf{0} & : & \textit{Inaction} \\ \alpha.P & : & \textit{Prefix } (\alpha \in \textit{Act}) \\ P + Q & : & \textit{Choice} \\ P \vee Q & : & \textit{Andor} \end{array}$$

where P and Q are already in $\mathcal{P}$. ∎

A Constant is a specification whose meaning is given by a defining equation. In fact, we assume that for every Constant $A \in \mathcal{K}$, there is a defining equation of the form $A \stackrel{\text{def}}{=} P$, where $P \in \mathcal{P}$. To avoid too many parentheses, combinators have binding power in the following order: Prefix > Choice > Andor.

We also define *ground specifications* P_0 with the following syntax:

$$P_0 \ ::= \ A_0 \mid \mathbf{0} \mid \alpha.P_0 \mid P_0 + P_0$$

where $A_0 \in \mathcal{K}_0 \subseteq \mathcal{K}$ and $\alpha \in \textit{Act}$. The set of ground specifications is denoted by $\mathcal{P}_0$. We assume that for every Constant $A_0 \in \mathcal{K}_0$, there is a defining equation of the form $A_0 \stackrel{\text{def}}{=} P_0$, where $P_0 \in \mathcal{P}_0$.

We use the following short notations:

$$\Sigma\{P_i : i \in I\} \equiv \begin{cases} \mathbf{0} & (I = \emptyset) \\ P_1 + P_2 + \cdots + P_n & (I = \{1, 2, \cdots, n\}) \end{cases}$$

$$\vee\{P_i : i \in I\} \equiv \begin{cases} \mathbf{0} & (I = \emptyset) \\ P_1 \vee P_2 \vee \cdots \vee P_n & (I = \{1, 2, \cdots, n\}) \end{cases}$$

Next, in order to define the multi-labeled transition system introduced in Section 1, an operator $\langle\ \rangle$ for any set S is defined as

$$\langle S \rangle = \{\langle e_1, e_2, \cdots, e_n \rangle : e_i \in S, n \geq 1\}.$$

$\langle S \rangle$ is called the *multi-set* of S. Three functions over a multi-set $\langle S \rangle$ are defined.

Definition 2.2 *Let $s, s' \in \langle S \rangle$, $e_i, e'_i \in S$, and $n, i \in \{1, 2, 3, \cdots\}$.*

- *$(\#s)$ is the length of s. Thus $\#\langle e_1, e_2, \cdots, e_n \rangle = n$.*
- *$(s \triangleleft i)$ is the i th element of s $(i \in [1, \#s])$. Thus $\langle e_1, e_2, \cdots, e_n \rangle \triangleleft i = e_i$.*
- *$(s; s')$ is the concatenation of s and s'.*
  Thus $\langle e_1, e_2, \cdots, e_n \rangle; \langle e'_1, e'_2, \cdots, e'_m \rangle = \langle e_1, e_2, \cdots, e_n, e'_1, e'_2, \cdots, e'_m \rangle$.

where $[1, n]$ is an abbreviation of an integer set $\{1, 2, \cdots, n\}$. ∎

Then we define multi-labeled transition systems (multi-LTS).

Definition 2.3 *A multi-LTS is a triple $(S, L, \rightarrow)$ where*

1. *S is a set of states,*
2. *L is a set of labels,*
3. *$\rightarrow$ is a set of transition relations such that*

$$\rightarrow \subseteq \{(e, u, s) : e \in S, u \in \langle L \rangle, s \in \langle S \rangle, \#u = \#s\}$$

We write $e \xrightarrow{u} s$ for $(e, u, s) \in \rightarrow$. ∎

The semantics of $SPEC^{\vee}$ is given by the multi-LTS $(\mathcal{P}, Act, \rightarrow)$, where $\rightarrow$ is defined in Definition 2.4. The set of *multi-specifications* $\langle\mathcal{P}\rangle$ is ranged over by $M, N, \cdots$ and the set of *multi-actions* $\langle Act\rangle$ is ranged over by $\mu, \nu, \cdots$.

Definition 2.4 *The transition relation $\rightarrow$ between a specification P, a multi-action μ, and a multi-specification M is the smallest relation satisfying the following inference rules. Each rule is to be read as follows: if the transition relation(s) above the line are inferred and the side condition(s) are satisfied, then the transition relation below the line can be also inferred.*

$$\textbf{Act}\ \frac{}{\alpha.P \xrightarrow{\langle\alpha\rangle} \langle P\rangle} \qquad \textbf{Choice}_1\ \frac{P \xrightarrow{\mu} M}{P+Q \xrightarrow{\mu} M}$$

$$\textbf{Con}\ \frac{P \xrightarrow{\mu} M}{A \xrightarrow{\mu} M}(A \stackrel{\text{def}}{=} P) \qquad \textbf{Choice}_2\ \frac{Q \xrightarrow{\mu} N}{P+Q \xrightarrow{\mu} N}$$

$$\textbf{Andor}\ \frac{P \xrightarrow{\mu} M \quad Q \xrightarrow{\nu} N}{P \vee Q \xrightarrow{\mu;\nu} M; N}$$

where ';' used in **Andor** *is the concatenation function given above.* ∎

3 Partial equalities

In this section two kinds of partial equalities in $SPEC^{\vee}$ are defined based on extented bisimulation relations.

First, we define a function for filtering actions.

Definition 3.1 *Filter function* ($/ : Act \times 2^{\mathcal{N}} \rightarrow Act$) *is defined as follows: For any $\alpha \in Act$ and $\Omega \subseteq \mathcal{N}$,*

$$\alpha/_{\Omega} = \begin{cases} \alpha & (\alpha \in \Omega) \\ \tau & (otherwise) \end{cases}$$

∎

If $\alpha \in \Omega$, then α is called a *valid action* for Ω, otherwise α is called an *invalid action* for Ω.

Next we define a function called *Selective function* to select the valid action from two actions observed through two filters. If two actions are inconsistent with each other, then Selective function returns the symbol $\perp$ $(\notin Act)$.

Definition 3.2 *Selective function $\bullet : Act \times 2^{\mathcal{N}} \times Act \times 2^{\mathcal{N}} \rightarrow Act \cup \{\perp\}$ is defined as follows: For any $\alpha_i \in Act$ and $\Omega_i \subseteq \mathcal{N}$,*

$$(\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \begin{cases} \alpha_1/_{\Omega_1} & (\alpha_1/_{\Omega_1} = \alpha_2/_{\Omega_2}) & (C1) \\ \alpha_1 & (\alpha_1 \in \Omega_1 - \Omega_2, \alpha_2 \notin \Omega_2) & (C2) \\ \alpha_2 & (\alpha_2 \in \Omega_2 - \Omega_1, \alpha_1 \notin \Omega_1) & (C3) \\ \perp & (otherwise) & (C4) \end{cases}$$

∎

$(C2)$ represents a selection of the valid action α_1. The condition $(\alpha_1 \in \Omega_1 - \Omega_2)$ of $(C2)$ represents that α_1 must be invalid for Ω_2, in spite of $\alpha_2 \notin \Omega_2$, because α_2 should be equal

to α_1 if $\alpha_1 \in \Omega_2$. $(C4)$ represents the three cases that (1) $\alpha_1 \in \Omega_1 \cap \Omega_2$ and $\alpha_1 \neq \alpha_2$ or (2) $\alpha_2 \in \Omega_1 \cap \Omega_2$ and $\alpha_1 \neq \alpha_2$ or (3) $\alpha_1 \in \Omega_1$, $\alpha_2 \in \Omega_2$, and $\alpha_1 \neq \alpha_2$.

For example, the following applications show properties of Selective function.

$$\begin{array}{l|l} (a,ab)\bullet(a,ac)=a \ \text{ by } (C1) & (b,ab)\bullet(e,ac)=b \ \text{ by } (C2) \\ (d,ab)\bullet(e,ac)=\tau \ \text{ by } (C1) & (a,ab)\bullet(\tau,ac)=\bot \ \text{ by } (C4) \\ (\tau,ab)\bullet(e,ac)=\tau \ \text{ by } (C1) & (b,ab)\bullet(c,ac)=\bot \ \text{ by } (C4) \end{array}$$

Then, (Ω_1,Ω_2)-*consistency* about actions is defined as follows. If two actions α_1 and α_2 observed through two filters Ω_1 and Ω_2, respectively, are consistent with each other by means of Selective function, then they are (Ω_1,Ω_2)-*consistent* and it is denoted by $(\alpha_1 \ {}_{\Omega_1}\doteq_{\Omega_2} \alpha_2)$, thus

$$ {}_{\Omega_1}\doteq_{\Omega_2} \ = \ \{(\alpha_1,\alpha_2) : (\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2) \neq \bot\}$$

We show several properties of Synthetic function.

Proposition 3.1 *For any* $\alpha_i \in Act$, $\Omega_i \subseteq \mathcal{N}$,

(1) *If* $(\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2)=\alpha$, *then* $(\alpha_1,\Omega_1)\bullet(\alpha,\Omega_{12})=\alpha$.
(2) *If* $(\alpha_1 \ {}_{\Omega_1}\doteq_{\Omega} \alpha)$,$(\alpha \ {}_{\Omega}\doteq_{\Omega_2} \alpha_2)$,*and* $\Omega_1 \subseteq \Omega$, *then* $(\alpha_1 \ {}_{\Omega_1}\doteq_{\Omega_2} \alpha_2)$.
(3) *If* $(\alpha_1 \ {}_{\Omega_1}\doteq_{\Omega} \alpha)$,$(\alpha \ {}_{\Omega}\doteq_{\Omega_2} \alpha_2)$,*and* $\Omega_{12} \subseteq \Omega$, *then* $(\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2)=\alpha/_{\Omega_{12}}$.
(4) $((\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2),\Omega_{12})\bullet(\alpha_3,\Omega_3)=(\alpha_1,\Omega_1)\bullet((\alpha_2,\Omega_2)\bullet(\alpha_3,\Omega_3),\Omega_{23})$

where $\Omega_{12}=\Omega_1\cup\Omega_2$ *and* $\Omega_{23}=\Omega_2\cup\Omega_3$. ∎

(1) means that the selected action α is consistent with α_1. (2) is conditional transitivity. (3) is useful for synthesis of two partial specifications. (4) means that the selected action does not depend on the order of the selections.

Now we define two kinds of partial equalities by using (Ω_1,Ω_2)-consistency.

Definition 3.3 *Let* $\Omega_1,\Omega_2 \subseteq \mathcal{N}$. *A binary relation* $\mathcal{S} \subseteq \mathcal{P}\times\mathcal{P}$ *over specifications is a* strong (Ω_1,Ω_2)-bisimulation, *if* $(P,Q)\in\mathcal{S}$ *implies that*

(i) *whenever* $P \xrightarrow{\mu} M$ *then, for some* N *and* ν, $Q \xrightarrow{\nu} N$ *and for some* i *and* j, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$,
(ii) *whenever* $Q \xrightarrow{\nu} N$ *then, for some* M *and* μ, $P \xrightarrow{\mu} M$ *and for some* i *and* j, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$. ∎

Definition 3.4 *Let* $\Omega_1,\Omega_2 \subseteq \mathcal{N}$. P *and* Q *are* strongly (Ω_1,Ω_2)-equivalent, *written* $P \ {}_{\Omega_1}\sim_{\Omega_2} Q$, *if* $(P,Q)\in\mathcal{S}$ *for some strong* (Ω_1,Ω_2)*-bisimulation* $\mathcal{S}$. ∎

Definition 3.5 *Let* $\Omega_1,\Omega_2 \subseteq \mathcal{N}$. *A binary relation* $\mathcal{S} \subseteq \mathcal{P}\times\mathcal{P}$ *over specifications is a* strong (Ω_1,Ω_2)-full-bisimulation, *if* $(P,Q)\in\mathcal{S}$ *implies that*

(i) *whenever* $P \xrightarrow{\mu} M$ *then, for some* N *and* ν, $Q \xrightarrow{\nu} N$ *and*
  (a) *for all* $i\in[1,\#\mu]$, *for some* j, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$,
  (b) *for all* $j\in[1,\#\nu]$, *for some* i, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$,
(ii) *whenever* $Q \xrightarrow{\nu} N$ *then, for some* M *and* μ, $P \xrightarrow{\mu} M$ *and*
  (a) *for all* $i\in[1,\#\mu]$, *for some* j, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$,
  (b) *for all* $j\in[1,\#\nu]$, *for some* i, $(\mu\triangleleft i \ {}_{\Omega_1}\doteq_{\Omega_2} \nu\triangleleft j)$ *and* $(M\triangleleft i, N\triangleleft j)\in\mathcal{S}$. ∎

Definition 3.6 *Let* $\Omega_1, \Omega_2 \subseteq \mathcal{N}$. P *and* Q *are* strongly (Ω_1, Ω_2)-full-equivalent, *written* $P \ {}_{\Omega_1}\simeq_{\Omega_2} Q$, *if* $(P, Q) \in \mathcal{S}$ *for some strong* (Ω_1, Ω_2)*-full-bisimulation* $\mathcal{S}$. ∎

The following relations clearly hold from the definitions.

- For any $P, Q \in \mathcal{P}$, $P \ {}_{\Omega_1}\simeq_{\Omega_2} Q \Rightarrow P \ {}_{\Omega_1}\sim_{\Omega_2} Q$
- For any $P_0, Q_0 \in \mathcal{P}_0$, $P_0 \ {}_{\Omega_1}\simeq_{\Omega_2} Q_0 \Leftrightarrow P_0 \ {}_{\Omega_1}\sim_{\Omega_2} Q_0$
- For any $P_0, Q_0 \in \mathcal{P}_0$, $P_0 \ {}_{\mathcal{N}}\sim_{\mathcal{N}} Q_0 \Leftrightarrow P_0 \sim Q_0$

where $\sim$ is strong equivalence defined in [1]. Although neither ${}_{\Omega_1}\sim_{\Omega_2}$ nor ${}_{\Omega_1}\simeq_{\Omega_2}$ is an equivalence relation, the following conditional reflexive, symmetric, and transitive laws hold for ${}_{\Omega_1}\sim_{\Omega_2}$.

Proposition 3.2 *For any* $P, Q \in \mathcal{P}$, $\Omega_i \subseteq \mathcal{N}$, *and* $R_0 \in \mathcal{P}_0$,

(1) $P \ {}_{\Omega_1}\sim_{\Omega_2} P$
(2) *If* $P \ {}_{\Omega_1}\sim_{\Omega_2} Q$, *then* $Q \ {}_{\Omega_2}\sim_{\Omega_1} P$
(3) *If* $P \ {}_{\Omega_1}\sim_{\Omega} R_0$, $R_0 \ {}_{\Omega}\sim_{\Omega_2} Q$, *and* $\Omega_1 \subseteq \Omega$, *then* $P \ {}_{\Omega_1}\sim_{\Omega_2} Q$. ∎

Proposition 3.2 also holds if ${}_{\Omega_1}\sim_{\Omega_2}$ is replaced to ${}_{\Omega_1}\simeq_{\Omega_2}$. Furthermore a medium R_0 of (3) does not have to be a *ground* specification for ${}_{\Omega_1}\simeq_{\Omega_2}$, thus for any $R \in \mathcal{P}$, if $P \ {}_{\Omega_1}\simeq_{\Omega} R$, $R \ {}_{\Omega}\simeq_{\Omega_2} Q$, and $\Omega_1 \subseteq \Omega$, then $P \ {}_{\Omega_1}\simeq_{\Omega_2} Q$.

4 Synthetic specification

In this section, we give a synthesis method of specifications preserving ${}_{\Omega_1}\sim_{\Omega_2}$. We explain how to synthesize specifications by using the following example.

$$\begin{array}{ll} A_c \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} & B_2 \equiv a.\tau.\mathbf{0} + \tau.\tau.\mathbf{0} + \tau.c.\mathbf{0} \\ B_1 \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.\tau.\mathbf{0} & B_3 \equiv \tau.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} \end{array} \qquad (*_2)$$

where $\equiv$ represents syntactic identity. B_1, B_2, and B_3 are partial specifications of the complete specification A_c, thus for any $i, j \in \{1, 2, 3\}$,

$$A_c \ {}_{\Omega}\sim_{\Omega_i} B_i \text{ and } B_i \ {}_{\Omega_i}\sim_{\Omega_j} B_j$$

where $\Omega = \{a, b, c\}$, $\Omega_1 = \{a, b\}$, $\Omega_2 = \{a, c\}$, and $\Omega_3 = \{b, c\}$.

At first, we consider *only* ground specifications for simplicity. A simple synthesis method is to use the following combinator.

$$\mathbf{SYN}\frac{Q_1 \xrightarrow{\langle\alpha_1\rangle} \langle Q_1'\rangle \quad Q_2 \xrightarrow{\langle\alpha_2\rangle} \langle Q_2'\rangle}{Q_1 \ {}_{\Omega_1}||_{\Omega_2} Q_2 \xrightarrow{\langle\alpha\rangle} \langle Q_1' \ {}_{\Omega_1}||_{\Omega_2} Q_2'\rangle}((\alpha_1, \Omega_1) \bullet (\alpha_2, \Omega_2) = \alpha)$$

This synthetic specification $Q_1 \ {}_{\Omega_1}||_{\Omega_2} Q_2$ performs valid actions of Q_1 and Q_2, but the following expected equation does not always hold.

$$Q_1 \ {}_{\Omega_1}||_{\Omega_2} Q_2 \ {}_{\Omega_{12}}\sim_{\Omega_i} Q_i \quad (i \in \{1, 2\}) \qquad (*_3)$$

where $\Omega_{12} = \Omega_1 \cup \Omega_2$. For the example $(*_2)$, the result of synthesis of B_1 and B_2 by ${}_{\Omega_1}||_{\Omega_2}$ is shown as follows:

$$B_1 \,{}_{ab}||_{ac} B_2 \sim a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + b.c.\mathbf{0} + \tau.\tau.\mathbf{0} + b.\mathbf{0} + \tau.\mathbf{0}$$

The last part $(b.\mathbf{0} + \tau.\mathbf{0})$ is important and breaks the expected equation $(*_3)$.

An easy method to guarantee $(*_3)$ may be to modify the combinator $\Omega_1||_{\Omega_2}$ as follows:

$$\mathbf{GLB}\frac{Q_1 \xrightarrow{\langle\alpha_1\rangle} \langle Q_1'\rangle \quad Q_2 \xrightarrow{\langle\alpha_2\rangle} \langle Q_2'\rangle}{Q_1 \,{}_{\Omega_1}[]_{\Omega_2} Q_2 \xrightarrow{\langle\alpha\rangle} \langle Q_1' \,{}_{\Omega_1}[]_{\Omega_2} Q_2'\rangle}\left(\begin{array}{l}(\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2)=\alpha,\\ Q_1' \;{}_{\Omega_1}\!\sim_{\Omega_2} Q_2'\end{array}\right),$$

but this definition circulates, because ${}_{\Omega_1}\!\sim_{\Omega_2}$ should be defined in terms of our transition relations. Therefore, a relation is defined instead of the combinator.

Definition 4.1 *Let $\Omega_1, \Omega_2 \subseteq \mathcal{N}$. A triadic relation $\mathcal{T}$ over ground specifications is a* strong (Ω_1, Ω_2)-GLB relation, *if $(P, Q_1, Q_2) \in \mathcal{T}$ implies that*

(i) whenever $P \xrightarrow{\langle\alpha\rangle} \langle P'\rangle$ then, for some Q_1', Q_2', α_1, and α_2,
$Q_1 \xrightarrow{\langle\alpha_1\rangle} \langle Q_1'\rangle$, $Q_2 \xrightarrow{\langle\alpha_2\rangle} \langle Q_2'\rangle$, $(\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2)=\alpha$, $Q_1' \;{}_{\Omega_1}\!\sim_{\Omega_2} Q_2'$, and $(P', Q_1', Q_2') \in \mathcal{T}$,
(ii) whenever $Q_1 \xrightarrow{\langle\alpha_1\rangle} \langle Q_1'\rangle$, $Q_2 \xrightarrow{\langle\alpha_2\rangle} \langle Q_2'\rangle$, $(\alpha_1,\Omega_1)\bullet(\alpha_2,\Omega_2)=\alpha \neq \perp$, and $Q_1' \;{}_{\Omega_1}\!\sim_{\Omega_2} Q_2'$,
then, for some P', $P \xrightarrow{\langle\alpha\rangle} \langle P'\rangle$ and $(P', Q_1', Q_2') \in \mathcal{T}$.

∎

Definition 4.2 *Let $\Omega_1, \Omega_2 \subseteq \mathcal{N}$. P is the strong (Ω_1, Ω_2)-GLB specification of Q_1 and Q_2, written $P \dot{\sim} (Q_1 \,{}_{\Omega_1}[]_{\Omega_2} Q_2)$, if $(P, Q_1, Q_2) \in \mathcal{T}$ for some strong (Ω_1, Ω_2)-GLB relation $\mathcal{T}$.* ∎

The strong (Ω_1, Ω_2)-GLB specification P of Q_1 and Q_2 is uniquely decided from Q_1 and Q_2 up to ${}_{\mathcal{N}}\!\sim_{\mathcal{N}}$, and the following relation holds.

$$\text{If } P \dot{\sim} (Q_1 \,{}_{\Omega_1}[]_{\Omega_2} Q_2) \text{ and } Q_1 \;{}_{\Omega_1}\!\sim_{\Omega_2} Q_2, \text{ then } P \;{}_{\Omega_1\cup\Omega_2}\!\sim_{\Omega_i} Q_i \; (i \in \{1,2\}) \quad (*_4)$$

For the example $(*_2)$, we can obtain the strong (ab, ac)-GLB specification A_{12} such that $A_{12} \dot{\sim} (B_1 \,{}_{ab}[]_{ac}\, B_2)$, considering Definition 4.2.

$$A_{12} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + b.c.\mathbf{0} + \tau.\tau.\mathbf{0}$$

Then $A_{12} \;{}_{abc}\!\sim_{ab} B_1$ and $A_{12} \;{}_{abc}\!\sim_{ac} B_2$ by $(*_4)$.

Unfortunately strong (Ω_1, Ω_2)-GLB specifications are not useful for synthesis of three or more specifications, namely stepwise refinement. For example, $(*_4)$ can not be used for the synthesis of A_{12} and B_3, because $A_{12} \;{}_{abc}\!\not\sim_{bc} B_3$.

The problem is that there are many synthetic specifications like A' satisfying $A' \;{}_{abc}\!\sim_{ab} B_1$ and $A' \;{}_{abc}\!\sim_{ac} B_2$. The complete specification A_c is one of them. Therefore it is expected to produce a comprehensive specification which includes all the synthetic specifications like A'. For the purpose a multi-LTS is needed and $SPEC^{\vee}$ is used.

Now we define principal strong (Ω_1, Ω_2)-synthetic specifications in $SPEC^{\vee}$.

Definition 4.3 *Let $\Omega_1, \Omega_2 \subseteq \mathcal{N}$. A triple relation $\mathcal{T} \subseteq \mathcal{P}^3$ over specifications is a* principal strong (Ω_1, Ω_2)-synthetic relation, *if $(P, Q_1, Q_2) \in \mathcal{T}$ implies that*

(i) for all M and μ, whenever $P \xrightarrow{\mu} M$ then,
(1) for some (N_1, ν_1), $Q_1 \xrightarrow{\nu_1} N_1$ and
(a) for all (N_2', ν_2', j', k'),
whenever $Q_2 \xrightarrow{\nu_2'} N_2'$, $\nu_1 \lhd j' \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2' \lhd k'$, and $N_1 \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \lhd k'$, then
for some i, $(\nu_1 \lhd j', \Omega_1) \bullet (\nu_2' \lhd k', \Omega_2) = \mu \lhd i$ and $(M \lhd i, N_1 \lhd j', N_2' \lhd k') \in \mathcal{T}$,
and
(b) for all $i' \in [1, \#\mu]$, for some (N_2', ν_2', j', k'), $N_1 \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \lhd k'$,
$Q_2 \xrightarrow{\nu_2'} N_2'$, $(\nu_1 \lhd j', \Omega_1) \bullet (\nu_2' \lhd k', \Omega_2) = \mu \lhd i'$, and $(M \lhd i', N_1 \lhd j', N_2' \lhd k') \in \mathcal{T}$,
or
(2) for some (N_2, ν_1), $Q_2 \xrightarrow{\nu_2} N_2$ and
(a) for all (N_1', ν_1', j', k'),
whenever $Q_1 \xrightarrow{\nu_1'} N_1'$, $\nu_1' \lhd j' \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2 \lhd k'$, and $N_1' \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2 \lhd k'$, then
for some i, $(\nu_1' \lhd j', \Omega_1) \bullet (\nu_2 \lhd k', \Omega_2) = \mu \lhd i$ and $(M \lhd i, N_1' \lhd j', N_2 \lhd k') \in \mathcal{T}$,
and
(b) for all $i' \in [1, \#\mu]$, for some (N_1', ν_1', j', k'), $N_1' \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2 \lhd k'$,
$Q_1 \xrightarrow{\nu_1'} N_1'$, $(\nu_1' \lhd j', \Omega_1) \bullet (\nu_2 \lhd k', \Omega_2) = \mu \lhd i'$, and $(M \lhd i', N_1' \lhd j', N_2 \lhd k') \in \mathcal{T}$,
and
(ii) for all $(N_1, N_2, \nu_1, \nu_2, j, k)$,
whenever $Q_1 \xrightarrow{\nu_1} N_1$, $Q_2 \xrightarrow{\nu_2} N_2$, $\nu_1 \lhd j \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2 \lhd k$, $N_1 \lhd j \ {}_{\Omega_1}\sim_{\Omega_2} N_2 \lhd k$, then
(1) for some (M, μ), $P \xrightarrow{\mu} M$ and
(a) for all (N_2', ν_2', j', k'),
whenever $Q_2 \xrightarrow{\nu_2'} N_2'$, $\nu_1 \lhd j' \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2' \lhd k'$, and $N_1 \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \lhd k'$, then
for some i, $(\nu_1 \lhd j', \Omega_1) \bullet (\nu_2' \lhd k', \Omega_2) = \mu \lhd i$ and $(M \lhd i, N_1 \lhd j', N_2' \lhd k') \in \mathcal{T}$,
and
(b) for all $i' \in [1, \#\mu]$, for some (N_2', ν_2', j', k'), $N_1 \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \lhd k'$,
$Q_2 \xrightarrow{\nu_2'} N_2'$, $(\nu_1 \lhd j', \Omega_1) \bullet (\nu_2' \lhd k', \Omega_2) = \mu \lhd i'$, and $(M \lhd i', N_1 \lhd j', N_2' \lhd k') \in \mathcal{T}$,
and
(2) for some (M, μ), $P \xrightarrow{\mu} M$ and
(a) for all (N_1', ν_1', j', k'),
whenever $Q_1 \xrightarrow{\nu_1'} N_1'$, $\nu_1' \lhd j' \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2 \lhd k'$, and $N_1' \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2 \lhd k'$, then
for some i, $(\nu_1' \lhd j', \Omega_1) \bullet (\nu_2 \lhd k', \Omega_2) = \mu \lhd i$ and $(M \lhd i, N_1' \lhd j', N_2 \lhd k') \in \mathcal{T}$,
and
(b) for all $i' \in [1, \#\mu]$, for some (N_1', ν_1', j', k'), $N_1' \lhd j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2 \lhd k'$,
$Q_1 \xrightarrow{\nu_1'} N_1'$, $(\nu_1' \lhd j', \Omega_1) \bullet (\nu_2 \lhd k', \Omega_2) = \mu \lhd i'$, and $(M \lhd i', N_1' \lhd j', N_2 \lhd k') \in \mathcal{T}$. ∎

Definition 4.4 *Let $\Omega_1, \Omega_2 \subseteq \mathcal{N}$. P is the principal strong (Ω_1, Ω_2)-synthetic specification of Q_1 and Q_2, written $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} Q_2)$, if $(P, Q_1, Q_2) \in \mathcal{T}$ for some principal strong (Ω_1, Ω_2)-synthetic relation $\mathcal{T}$.* ∎

Definition 4.3 bases on the idea of Definition 4.1, thus selection of valid actions and preservation of ${}_{\Omega_1}\sim_{\Omega_2}$.

Then the expected relation is obtained.

Proposition 4.1 *Let* $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$, $Q_1 \ {}_{\Omega_1}\sim_{\Omega_2} \ Q_2$, *and* $\Omega_{12} = \Omega_1 \cup \Omega_2$. *For any ground specifications* $P_0 \in \mathcal{P}_0$,

$$P_0 \ {}_{\Omega_{12}}\sim_{\Omega_{12}} P \quad \textbf{iff} \quad P_0 \ {}_{\Omega_{12}}\sim_{\Omega_1} Q_1 \textit{ and } P_0 \ {}_{\Omega_{12}}\sim_{\Omega_2} Q_2.$$

Proof

($\Rightarrow$) We show that the following $\mathcal{S}$ is a strong (Ω_{12}, Ω_1)-bisimulation.

$$\mathcal{S} = \{(P_0, Q_1) : P_0 \in \mathcal{P}_0, \exists P, Q_2 \in \mathcal{P}, P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2), Q_1 \ {}_{\Omega_1}\sim_{\Omega_2} \ Q_2, P_0 \ {}_{\Omega_{12}}\sim_{\Omega_{12}} P\}$$

Let $(P_0, Q_1) \in \mathcal{S}$. Thus, P_0 is a ground specification, and for some P and Q_2, $Q_1 \ {}_{\Omega_1}\sim_{\Omega_2} \ Q_2$, $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_{12}} P$, and $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$.

- (i) Let $P_0 \xrightarrow{\langle\alpha\rangle} \langle P_0' \rangle$. Since $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_{12}} P$, for some M and μ, we have that $P \xrightarrow{\mu} M$, for some i, $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_{12}} \ \mu \triangleleft i)$ and $P_0' \ {}_{\Omega_{12}}\sim_{\Omega_{12}} M \triangleleft i$. This implies the following either case.
  - By Definition 4.3(i)(1)(b), since $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$, for some N_1 and ν_1, we have $Q_1 \xrightarrow{\nu_1} N_1$ and for some N_2, ν_2, j, and k, $Q_2 \xrightarrow{\nu_2} N_2$, $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \mu \triangleleft i$, $N_1 \triangleleft j \ {}_{\Omega_1}\sim_{\Omega_2} \ N_2 \triangleleft k$, and $M \triangleleft i \doteq (N_1 \triangleleft j \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ N_2 \triangleleft k)$. Thus $(P_0', N_1 \triangleleft j) \in \mathcal{S}$, Here, by Proposition 3.1(1), $(\nu_1 \triangleleft j \ {}_{\Omega_1}\dot{=}_{\Omega_{12}} \ \mu \triangleleft i)$. Hence, by Proposition 3.1(2), $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_1} \ \nu_1 \triangleleft j)$.
  - The case by Definition 4.3(i)(2)(b) can be shown by the same argument as the above case.

- (ii) Let $Q_1 \xrightarrow{\nu_1} N_1$. Since $Q_1 \ {}_{\Omega_1}\sim_{\Omega_2} \ Q_2$, for some N_2 and ν_2, we have that $Q_2 \xrightarrow{\nu_2} N_2$, for some j and k, $(\nu_1 \triangleleft j \ {}_{\Omega_1}\dot{=}_{\Omega_2} \ \nu_2 \triangleleft k)$ and $N_1 \triangleleft j \ {}_{\Omega_1}\sim_{\Omega_2} \ N_2 \triangleleft k$. By Definition 4.3(ii)(1), since $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$, for some M and μ, we have $P \xrightarrow{\mu} M$. Since $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_{12}} P$, for some P_0' and α, we have that $P_0 \xrightarrow{\langle\alpha\rangle} \langle P_0' \rangle$, for some $i \in [1, \#\mu]$, $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_{12}} \ \mu \triangleleft i)$ and $P_0' \ {}_{\Omega_{12}}\sim_{\Omega_{12}} M \triangleleft i$. By Definition 4.3(ii)(1)(b), for some N_2' and ν_2', j', and k', $Q_2 \xrightarrow{\nu_2'} N_2'$, $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu_2' \triangleleft k', \Omega_2) = \mu \triangleleft i$, $N_1 \triangleleft j' \ {}_{\Omega_1}\sim_{\Omega_2} \ N_2' \triangleleft k'$, and $M \triangleleft i \doteq (N_1 \triangleleft j' \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ N_2' \triangleleft k')$. Thus, $(P_0', N_1 \triangleleft j') \in \mathcal{S}$. Here, by Proposition 3.1(1), $(\nu_1 \triangleleft j' \ {}_{\Omega_1}\dot{=}_{\Omega_{12}} \ \mu \triangleleft i)$. Hence, by Proposition 3.1(2), $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_1} \ \nu_1 \triangleleft j')$.

($\Leftarrow$) We show that the following $\mathcal{S}$ is a strong $(\Omega_{12}, \Omega_{12})$-bisimulation.

$$\mathcal{S} = \{(P_0, P) : P_0 \in \mathcal{P}_0, \exists Q_1, Q_2 \in \mathcal{P}, P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2), P_0 \ {}_{\Omega_{12}}\sim_{\Omega_1} \ Q_1, P_0 \ {}_{\Omega_{12}}\sim_{\Omega_2} \ Q_2\}$$

Let $(P_0, P) \in \mathcal{S}$. Thus, P_0 is a ground specification, and for some Q_1 and Q_2, $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_1} \ Q_1$, $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_2} \ Q_2$, and $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$.

- (i) Let $P_0 \xrightarrow{\langle\alpha\rangle} \langle P_0' \rangle$. For each $n \in \{1, 2\}$, since $P_0 \ {}_{\Omega_{12}}\sim_{\Omega_n} \ Q_n$, for some N_n and ν_n, we have that $Q_n \xrightarrow{\nu_n} N_n$, for some j and k, $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_1} \ \nu_1 \triangleleft j)$, $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_2} \ \nu_2 \triangleleft k)$, $P_0' \ {}_{\Omega_{12}}\sim_{\Omega_1} \ N_1 \triangleleft j$, and $P_0' \ {}_{\Omega_{12}}\sim_{\Omega_2} \ N_2 \triangleleft k$. By Proposition 3.1(2), $(\nu_1 \triangleleft j \ {}_{\Omega_1}\dot{=}_{\Omega_2} \ \nu_2 \triangleleft k)$. By Proposition 3.2(3), $N_1 \triangleleft j \ {}_{\Omega_1}\sim_{\Omega_2} \ N_2 \triangleleft k$. Thus, by Definition 4.3(ii)(1)(a), since $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$, for some M and μ, we have $P \xrightarrow{\mu} M$ and for some i, $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \mu \triangleleft i$ and $M \triangleleft i \doteq (N_1 \triangleleft j \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ N_2 \triangleleft k)$. Thus $(P_0', M \triangleleft i) \in \mathcal{S}$. Here, by Proposition 3.1(3), $(\nu_1 \triangleleft j, \Omega_1) \bullet (\nu_2 \triangleleft k, \Omega_2) = \alpha/_{\Omega_{12}}$. Hence, $(\alpha \ {}_{\Omega_{12}}\dot{=}_{\Omega_{12}} \ \mu \triangleleft i)$, because $\alpha/_{\Omega_{12}} = \mu \triangleleft i$.

- (ii) Let $P \xrightarrow{\mu} M$. This implies the following either case.

- By Definition 4.3(i)(1), since $P \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2)$, for some N_1 and ν_1, we have $Q_1 \xrightarrow{\nu_1} N_1$. Since $P_0 \ {}_{\Omega_{12}}\!\sim_{\Omega_1} Q_1$, for some α and P_0', we have that $P_0 \xrightarrow{\langle\alpha\rangle} \langle P_0' \rangle$, for some j, $(\alpha \ {}_{\Omega_{12}}\!\doteq_{\Omega_1} \nu_1 \lhd j)$ and $P_0' \ {}_{\Omega_{12}}\!\sim_{\Omega_1} N_1 \lhd j$. Since $P_0 \ {}_{\Omega_{12}}\!\sim_{\Omega_2} Q_2$, for some N_2 and ν_2, we have that $Q_2 \xrightarrow{\nu_2} N_2$, for some k, $(\alpha \ {}_{\Omega_{12}}\!\doteq_{\Omega_2} \nu_2 \lhd k)$ and $P_0' \ {}_{\Omega_{12}}\!\sim_{\Omega_2} N_2 \lhd k$. By Proposition 3.1(3), $(\nu_1 \lhd j, \Omega_1) \bullet (\nu_2 \lhd k, \Omega_2) = \alpha/_{\Omega_{12}}$. By Proposition 3.2(3), $N_1 \lhd j \ {}_{\Omega_1}\!\sim_{\Omega_2} N_2 \lhd k$. By Definition 4.3(i)(1)(a), for some i, $(\nu_1 \lhd j, \Omega_1) \bullet (\nu_2 \lhd k, \Omega_2) = \mu \lhd i$ and $M \lhd i \doteq (N_1 \lhd j \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} N_2 \lhd k)$. Thus, $(P_0', M \lhd i) \in \mathcal{S}$ and $(\alpha \ {}_{\Omega_{12}}\!\doteq_{\Omega_{12}} \mu \lhd i)$.
- The case by Definition 4.3(i)(2) can be shown by a symmetric argument with the above case. ∎

Next, we give two important propositions. Proposition 4.2 shows that the principal strong (Ω_1, Ω_2)-synthetic specification P of two specifications Q_1 and Q_2 is uniquely decided from Q_1 and Q_2 up to ${}_{\mathcal{N}}\!\simeq_{\mathcal{N}}$.

Proposition 4.2 *Let* $P_{12} \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2)$. *For any specification* $P \in \mathcal{P}$,

$$P_{12} \ {}_{\mathcal{N}}\!\simeq_{\mathcal{N}} P \quad \textbf{iff} \quad P \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2).$$

Proof We show that the following $\mathcal{T}$ is a principal strong (Ω_1, Ω_2)-synthetic relation and $\mathcal{S}$ is a strong $(\mathcal{N}, \mathcal{N})$-full-bisimulation.

$$\mathcal{T} = \{(P, Q_1, Q_2) : \exists P_{12} \in \mathcal{P}, P_{12} \ {}_{\mathcal{N}}\!\simeq_{\mathcal{N}} P, P_{12} \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2)\}$$
$$\mathcal{S} = \{(P_{12}, P) : \exists Q_1, Q_2 \in \mathcal{P}, P \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2), P_{12} \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2)\}$$

The proof is omitted because of lack of space. ∎

Proposition 4.3 shows how to produce the principal strong (Ω_1, Ω_2)-synthetic specification of Q_1 and Q_2. In fact, $\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2)$ can be efficiently produced from Q_1 and Q_2, if Q_1 and Q_2 have finite states.

Proposition 4.3 $\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2)$ *is defined as follows:*

$$\begin{aligned}\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2) \equiv \quad & \Sigma\{\mathsf{V}\{\alpha.\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1', Q_2') : (\alpha, Q_1', Q_2') \in E^{\Omega_1}_{\Omega_2}(\nu_1, N_1, Q_2)\} \\ & \quad : (\nu_1, N_1) \in D(Q_1)\} \\ & + \Sigma\{\mathsf{V}\{\alpha.\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1', Q_2') : (\alpha, Q_2', Q_1') \in E^{\Omega_1}_{\Omega_2}(\nu_2, N_2, Q_1)\} \\ & \quad : (\nu_2, N_2) \in D(Q_2)\}\end{aligned}$$

where

$$D(P) = \{(\mu, M) : P \xrightarrow{\mu} M\}$$
$$E^{\Omega_1}_{\Omega_2}(\mu, M, Q) = \{(\alpha, P', Q') : \exists (\nu, N, i, j), Q \xrightarrow{\nu} N,\ (\mu \lhd i, \Omega_1) \bullet (\nu \lhd j, \Omega_2) = \alpha,\ M \lhd i \ {}_{\Omega_1}\!\sim_{\Omega_2} N \lhd j,\ P' \equiv M \lhd i,\ Q' \equiv N \lhd j\}$$

Then, $\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2) \doteq (Q_1 \ {}_{\Omega_1}\!\bigcap\!{}_{\Omega_2} Q_2)$.

Proof We show that the following $\mathcal{T}$ is a principal strong (Ω_1, Ω_2)-synthetic relation.

$$\mathcal{T} = \{(\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2), Q_1, Q_2) : Q_1, Q_2 \in \mathcal{P}\}$$

Let $(P, Q_1, Q_2) \in \mathcal{T}$. Thus $P \equiv \mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2)$.

(i) Let $\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2) \xrightarrow{\mu} M$. By $\mathbf{Choice_{1,2}}$, it implies the following either case (1) or (2).

(1) For some $(\nu_1, N_1) \in D(Q_1)$,

$$\mathsf{V}\{\alpha.\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1', Q_2') : (\alpha, Q_1', Q_2') \in E^{\Omega_1}_{\Omega_2}(\nu_1, N_1, Q_2)\} \xrightarrow{\mu} M.$$

We have $Q_1 \xrightarrow{\nu_1} N_1$, because $(\nu_1, N_1) \in D(Q_1)$.

- (a) Let N_2', ν_2', j', and k' such that $Q_2 \xrightarrow{\nu_2'} N_2'$, $\nu_1 \triangleleft j' \ {}_{\Omega_1}\doteq_{\Omega_2} \nu_2' \triangleleft k'$, and $N_1 \triangleleft j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \triangleleft k'$. In this case, $(\alpha, N_1 \triangleleft j', N_2' \triangleleft k') \in E^{\Omega_1}_{\Omega_2}(\nu_1, N_1, Q_2)$, where $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu_2' \triangleleft k', \Omega_2) = \alpha$. Thus, by **Andor** and **Act**, for some $i \in [1, \#\mu]$, we have $\mu \triangleleft i = \alpha$ and $M \triangleleft i \equiv \mathcal{SPS}^{\Omega_1}_{\Omega_2}(N_1 \triangleleft j', N_2' \triangleleft k')$. Hence, $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu_2' \triangleleft k', \Omega_2) = \alpha = \mu \triangleleft i$ and $(M \triangleleft i, N_1 \triangleleft j', N_2' \triangleleft k') \in \mathcal{T}$.

- (b) Let $i \in [1, \#\mu]$. By **Andor** and **Act**, for some $(\alpha, Q_1', Q_2') \in E^{\Omega_1}_{\Omega_2}(\nu_1, N_1, Q_2)$, we have $\mu \triangleleft i = \alpha$ and $M \triangleleft i \equiv \mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1', Q_2')$. Thus, for some N_2', ν_2', j', and k', we have $Q_2 \xrightarrow{\nu_2'} N_2'$, $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu_2' \triangleleft k', \Omega_2) = \alpha$, $N_1 \triangleleft j' \ {}_{\Omega_1}\sim_{\Omega_2} N_2' \triangleleft k'$, $Q_1' \equiv N_1 \triangleleft j'$, and $Q_2' \equiv N_2' \triangleleft k'$, because $(\alpha, Q_1', Q_2') \in E^{\Omega_1}_{\Omega_2}(\nu_1, N_1, Q_2)$. Hence, $(\nu_1 \triangleleft j', \Omega_1) \bullet (\nu_2' \triangleleft k', \Omega_2) = \alpha = \mu \triangleleft i$ and $(M \triangleleft i, N_1 \triangleleft j', N_2' \triangleleft k') \in \mathcal{T}$.

(2) For some $(\nu_2, N_2) \in D(Q_2)$,

$$\bigvee\{\alpha.\mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1', Q_2') : (\alpha, Q_2', Q_1') \in E^{\Omega_1}_{\Omega_2}(\nu_2, N_2, Q_1)\} \xrightarrow{\mu} M.$$

This case is symmetric with the case (i)(1).

(ii) This case can be shown by a similar argument to the case (i). ∎

We can check that $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$ by finding a principal strong (Ω_1, Ω_2)-synthetic relation $\mathcal{T}$ such that $(P, Q_1, Q_2) \in \mathcal{T}$, but Definition 4.3 is not easy. Proposition 4.2 and 4.3 imply another method for the check. Thus, we can check that $P \doteq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$ by checking that $P \ {}_{\mathcal{N}}\simeq_{\mathcal{N}} \mathcal{SPS}^{\Omega_1}_{\Omega_2}(Q_1, Q_2)$.

Finally a theorem for stepwise synthesizing n specifications is given.

Theorem 4.4 *Assume that $n \geq 1$, for any ground specification $P_0 \in \mathcal{P}_0$,*

$$P_0 \ {}_{\Omega}\sim_{\Omega} P_n \quad \textbf{iff} \quad \textit{for any } i \in [1, n], P_0 \ {}_{\Omega}\sim_{\Omega_i} Q_i,$$

$P_n \ {}_{\Omega}\sim_{\Omega_{n+1}} Q_{n+1}$, $P_{n+1} \doteq (P_n \ {}_{\Omega}\lceil\rceil_{\Omega_{n+1}} \ Q_{n+1})$, and $\Omega_i \subseteq \Omega$ $(i \in [1, n])$.

Then, for any ground specification $P_0 \in \mathcal{P}_0$,

$$P_0 \ {}_{\Omega'}\sim_{\Omega'} P_{n+1} \quad \textbf{iff} \quad \textit{for any } i \in [1, n+1], P_0 \ {}_{\Omega'}\sim_{\Omega_i} Q_i$$

where $\Omega' = \Omega \cup \Omega_{n+1}$.

Proof This result can be easily shown by Proposition 4.1. ∎

In Theorem 4.4, if $P_n \ {}_{\Omega}\not\sim_{\Omega_{n+1}} Q_{n+1}$, then there is no ground specification P_0 such that $P_0 \ {}_{\Omega'}\sim_{\Omega_i} Q_i$ for any $i \in [1, n+1]$ by Proposition 3.2(3). Therefore $P_n \ {}_{\Omega}\sim_{\Omega_{n+1}} Q_{n+1}$ is needed.

For example, Theorem 4.4 is used for the following situations.

1. Reconstruction of the complete ground specification P_0 from many partial specifications Q_i $(i \in I)$, such that $P_0 \ {}_{\Omega}\sim_{\Omega_i} Q_i$ and $\bigcup_{i \in I} \Omega_i = \Omega$. We produce a specification P_I by recursively using Theorem 4.4 and Proposition 3.2(3) like $P_I \ {}_{\Omega}\sim_{\Omega} P_0$ and $P_I \ {}_{\Omega}\sim_{\Omega_i} Q_i$.

2. Refinement of a specification P_n, by synthesizing a new requirement Q_{n+1}. Theorem 4.4 guarantees that the refined specification P_{n+1} satisfys all the requirements Q_i which P_n satisfys, thus preservation of the requirements.

Finally we apply our synthesis method to the example $(*_2)$. By Proposition 4.3, the principal strong (ab, ac)-synthetic specification $A_{12}^{\sqcap}$ of B_1 and B_2 is produced as follows:

$$A_{12}^{\sqcap} \equiv \mathcal{SPS}_{ac}^{ab}(B_1, B_2) \equiv a.\tau.\mathbf{0} + (b.\tau.\mathbf{0} \vee b.c.\mathbf{0}) + (\tau.c.\mathbf{0} \vee b.c.\mathbf{0}) + (b.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (\tau.c.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

Proposition 4.1 guarantees that $A_{c\ abc} \sim_{abc} A_{12}^{\sqcap}$, because $A_{c\ abc} \sim_{ab} B_1$ and $A_{c\ abc} \sim_{ac} B_2$. Furthermore by Proposition 4.3, the principal strong (abc, bc)-synthetic specification $A_{123}^{\sqcap}$ of $A_{12}^{\sqcap}$ and B_3 is produced as follows:

$$A_{123}^{\sqcap} \equiv \mathcal{SPS}_{ac}^{abc}(A_{12}^{\sqcap}, B_3) \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + (a.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (b.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0}) + (\tau.c.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

By Proposition 3.2(3), we have that $A_{12}^{\sqcap}\ {}_{abc}\!\sim_{bc} B_3$. Thus, by Theorem 4.4, for any ground specification A_0,

$$A_0\ {}_{abc}\!\sim_{abc} A_{123}^{\sqcap} \quad \textbf{iff} \quad \text{for any } i \in [1,3], A_0\ {}_{abc}\!\sim_{\Omega_i} B_i. \qquad (*_5)$$

The following two specifications are all the ground specifications A_0 up to ${}_{abc}\!\sim_{abc}$ such that $A_0\ {}_{abc}\!\sim_{abc} A_{123}^{\sqcap}$.

(1) $A_{01} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0}$

(2) $A_{02} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + \tau.\tau.\mathbf{0}$

The first specification A_{01} corresponds to the complete specification A_c. The second specification A_{02} is also a candidate satisfying B_1, B_2, and B_3.

5 Related work

Decomposition and refinement methods of specifications have been proposed, for example in [2, 3]. In [2], algorithms to decompose a specification into two parallel processes are presented in LOTOS. The algorithms are useful for implementation. Our strong (Ω_1, Ω_2)-equivalence is used for extracting partial specification about interesting actions, thus the purpose is different from one of [2].

In [3], a stepwise refinement of specifications is proposed based on a new parallel combinator in LOTOS. The synchronous rule of the combinator is

$$\textbf{Sync}\frac{Q_1 \xrightarrow{\mu_1} Q_1' \quad Q_2 \xrightarrow{\mu_2} Q_2'}{Q_1|_A Q_2 \xrightarrow{\mu_1 \cup \mu_2} Q_1'|_A Q_2'} \left(\mu_1 \cap \mu_2 = \mu_1 \cap A = \mu_2 \cap A \right).$$

The labels μ on the transitions are sets of actions $\{a_1, \cdots, a_n\}$. It increases greatly the simplicity and modularity of refined specifications. Our refinement intuitively bases on **GLB** rule introduced in Section 4. The most important difference between **GLB** and **Sync** is that **GLB** has the condition $Q_1'\ {}_{\Omega_1}\!\sim_{\Omega_2} Q_2'$. The condition is available for specifications with indeterminate choices and produces a refined specification P such that $P\ {}_{\Omega_1 \cup \Omega_2}\!\sim_{\Omega_i} Q_i$.

In order to partially analyze processes, we already proposed a process algebra *CCSG*[4]. In CCSG we approximately analyze processes by neglecting unimportant distant actions. A disadvantage of CCSG is used for only specific systems, where actions has information about the importance and the position. $SPEC^{\vee}$ presented in this paper is widely applicable to models based on LTS.

6 Conclusion

In this paper we have presented strong (Ω_1, Ω_2)-equivalence ${}_{\Omega_1}\sim_{\Omega_2}$ to relate partial specifications and have presented principal strong (Ω_1, Ω_2)-synthetic specifications to synthesize partial specifications preserving ${}_{\Omega_i}\sim_{\Omega_2}$. Theorem 4.4 is used for stepwise refinement of specifications and it guarantees that a new requirement does not break previous requirements. If P is the principal strong (Ω_1, Ω_2)-synthetic specifications of Q_1 and Q_2, then all the ground specifications included in P satisfy both Q_1 and Q_2, and they are all the ground specifications satisfying both Q_1 and Q_2.

We have future works as follows:

1. The most important future work is to develop a *weak* version of principal *strong* (Ω_1, Ω_2)-synthetic specifications, by neglecting internal actions as far as possible. Interesting examples will be shown by using the weak version.

2. We should clarify more properties of principal *strong* (Ω_1, Ω_2)-synthetic specifications. For example, it is expected that Definition 4.3 of principal strong (Ω_1, Ω_2)-synthetic specifications is slightly weakened, because there are synthetic specifications P such that, for any ground specifications P_0,

   $P_0 \ {}_{\Omega_1\cup\Omega_2}\sim_{\Omega_1\cup\Omega_2} P$ **iff** $P_0 \ {}_{\Omega_1\cup\Omega_2}\sim_{\Omega_1} Q_1$ and $P_0 \ {}_{\Omega_1\cup\Omega_2}\sim_{\Omega_2} Q_2$

   and $P \not\simeq (Q_1 \ {}_{\Omega_1}\lceil\rceil_{\Omega_2} \ Q_2)$. The example $(*_2)$ in Section 4 is used here again. The following specification $A'^{\sqcap}_{123}$ satisfys $(*_5)$, but $A'^{\sqcap}_{123} \not\simeq (A^{\sqcap}_{12} \ {}_{abc}\lceil\rceil_{bc} \ B_3)$.

   $$A'^{\sqcap}_{123} \equiv a.\tau.\mathbf{0} + b.\tau.\mathbf{0} + \tau.c.\mathbf{0} + (a.\tau.\mathbf{0} \vee \tau.\tau.\mathbf{0})$$

3. We will develop a verification tool for strong (Ω_1, Ω_2)-equivalence such as the concurrency workbench [5] and a synthesis tool for producing principal strong (Ω_1, Ω_2)-synthesis specifications.

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